Analysis of trapped surfaces in higher dimensional dust collapse

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In the present work we analyze the dynamics of apparent horizon in higher dimensional (HD) dust collapse. For this study we have taken in to consideration the two different types of dust models. We propose the concept of 'trapped range' of initial data in the different higher dimensional spacetimes. We show that 'trapped range' of initial data increases with the increase in dimensions of the spacetimes.

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I. INTRODUCTION

Today, a well known and challenging open problem in general relativity is the cosmic censorship conjecture (CCH) [1]. It states that, for physically reasonable initial data, spacetime cannot admit naked singularities (NS). In other words if a singularity forms in the gravitational collapse, then it must be covered by an event horizon of the gravity. Despite several attempts by many researchers neither general proof nor precise mathematical formulation of the conjecture has been available so far. On the contrary, some solutions of the Einstein equation with physically reasonable initial data evolving in to naked singularities have been found [2].

There has been extensive research work on naked singularities and black holes and in particular on whether or not cosmic censorship conjecture (CCH) is true [3]. A classical statement in Hawking and Ellis [4] assert that the boundary of a trapped region is an apparent horizon. It is known that apparent horizon forms in the region of sufficiently strong gravitational field. An apparent horizon seems to play an important role in deciding the nature of the singularity. Much literature on apparent horizon have been appeared so far [5]. It is indeed believed that formation of the central singularity earlier than apparent horizon is necessary condition for a singularity to be naked. A singularity cannot be naked if it occurs after the formation of the apparent horizon. Jhingan el.al. [6] has shown that absence of apparent horizon formation prior to the central singularity does not necessarily imply nakedness. In the present work we generalize this result to *(N+2)*-dimensional spacetimes. To this end, we have chosen Tolman-Bondi model, which is the simplest solution to the Einstein equation, which admits both naked singularity as well as a black hole (BH) depending upon the nature of the initial data.

The possible existent of large extra dimensions has opened up new and exciting directions of research in quantum gravity [7]. Hence to prove or disprove cosmic censorship conjecture it becomes necessary to study the gravitational collapse in the higher dimensions as well. Many papers on higher dimensional collapse have been appeared so far [8], which show the existence of both black holes and naked singularities depending upon the nature of the initial data. Thus initial data plays a vital role in deciding the nature of the singularity in the gravitational collapse of a massive star.

The present paper is organized as follows:-

In Sec. II, we consider the class of Tolman-Bondi models which are generally non-self similar, but they can be reduced to self-similar under certain condition. In this Sec. we discuss the dynamics of an apparent horizon in the higher dimensional space times.

In Sec. III, we analyze the formation of apparent horizon in the gravitational collapse from finitely differentiable initial data. We conclude with some concluding remarks in Sec.IV.

II. SPHERICALLY SYMMETRIC DUST COLLAPSE IN HIGHER DIMENSIONAL SPACETIME

Let us consider the metric for *(N+2)*–dimensional spacetime with spherical symmetry [9]

$$ds^2 = -dt^2 + \frac{R'^2 dr^2}{1+f(r)} + R^2 d\Omega^2 \quad , \tag{1}$$

where

$$d\Omega^2 = d\theta_1{}^2 + Sin^2\theta_1 d\theta_2{}^2 + Sin^2\theta_1 Sin^2\theta_2 d\theta_3{}^2 + ...Sin^2\theta_1 Sin^2\theta_2 ...Sin^2\theta_{N-1} d\theta^2{}_N \ , \tag{2}$$

is the metric on *N*-sphere.

Energy momentum tensor for dust has the form

$$T_{ab} = \varepsilon(t,r)\delta^t_a \delta^t_b \ , \tag{3}$$

where $u_a = \delta_a^t$ is the *(N+2)*-dimensional velocity, *R* is the area radius at time *t* of the shell having the comoving coordinate *r*.

The Einstein Eqs. for the collapsing cloud are

$$\varepsilon(r,t) = \frac{NF'}{2R^N R'} \tag{4}$$

and

$$\dot{R}^2 = \frac{F(r)}{R^{N-1}} + f(r) \ . \tag{5}$$

(we have set up $\frac{8\pi G}{c^4} = 1$.)

For simplicity, we consider the marginally bound case $f(r) = 0$.

In the collapsing case , Eq. (5) yields

$$\dot{R} = \frac{-\sqrt{F(r)}}{R^{(N-1)/2}} \quad . \tag{6}$$

Integrating Eq. (6) we get

$$R^{(N+1)/2} = r^{(N+1)/2} - \frac{(N+1)}{2}\sqrt{F}t \ , \tag{7}$$

where we have used the freedom in the scaling of the comoving coordinate *r* to set up *R(0,r)* = *r* at the starting epoch of the collapse.

We consider $t_s(r)$ as the time at which area radius *R* become zero, hence Eq. (7) yields

$$t_s(r) = \left(\frac{2}{N+1}\right)\frac{r^{(N+1)/2}}{\sqrt{F}} \quad . \tag{8}$$

Physically, $t_s(r)$ is the comoving time at which the shell of matter labeled by *r* becomes singular. As the density increase unboundedly, trapped surfaces has to form within the collapsing cloud. The outmost boundary of these trapped surfaces is known as apparent horizon.

In *(N+2)*-dimensional spacetime it follows from Eq. (6) that apparent horizon is given by

$$R(t_{ah}(r), r) = F^{1/(N-1)} \quad , \tag{9}$$

where $t_{ah}(r)$ is the time at which apparent horizon forms.

Inserting *R* from Eq.(9) into Eq.(7) we get

$$t_{ah}(r) = \left(\frac{2}{N+1}\right)\frac{r^{(N+1)/2}}{\sqrt{F}} - \frac{2}{N+1}F^{1/(N-1)} \quad . \tag{10}$$

Equation (10) determine the behavior of the apparent horizon in the vicinity of the central singularity in *(N+2)*-dimensional space time.

First we consider the class of TBL models which are generally non self-similar, but they can be reduced to self-similar under certain condition. In case of 4 dimensional space time this type of solution has been studied in Ref.[10].

Mass function in general *(N+2)*-dimensional spacetime for this class of model is given by [11].

$$F(r) = \lambda(r) r^{(N-1)} \quad , \quad \lambda(0) = \lambda_0 > 0(finite) \, . \tag{11}$$

It should be noted that the space time becomes self-similar when one keep $\lambda(r)$= *constant* and *f(r)* = *Const.* With the choice of above mass function it can be seen that the density of the space time is inversely proportional to t^2 and hence is finite on the initial epoch $t = t_i < 0$ [10, 11].

Inserting above mass function $F(r)$ into Eq. (10) we obtain

$$t_{ah}(r) = \frac{2}{N+1}\left[\frac{1}{\sqrt{\lambda}} - \lambda^{1/(N-1)}\right]r\ . \tag{12}$$

Since $\lambda(0) \neq 0$, it can be observed that $t_s(0) = 0$. Hence it follows that the point $r=0,\ t=0$ corresponds to the central singularity on the hypersurface $t = 0$, where the energy density becomes infinite. Since in these classes of models the central singularity occurs at $t=0$, we can write

$$t_{ah}(r) = t_{ah}(r) - t_s(0) = \frac{2}{N+1}\left[\frac{1}{\sqrt{\lambda}} - \lambda^{1/(N-1)}\right]r\ . \tag{13}$$

From above equation it is clear that

$t_s(0) < t_{ah}(r)$ if

$$\frac{1}{\sqrt{\lambda}} - \lambda^{1/(N-1)} > 0\ , \tag{14}$$

which in turns reduces to

$$\lambda^{(N+1)/2(N-1)} < 1\ . \tag{15}$$

Hence the central singularity forms earlier than apparent horizon if

$$\lambda_0^{(N+1)/2(N-1)} < 1 \tag{16}$$

i.e. if

$$\lambda_0 < 1\ . \tag{17}$$

Thus in any $(N+2)$-dimensional spacetime, if $\lambda_0<1$, then the central shell focusing singularity could be naked. In the present work, we are not concerning about the local or global nakedness of the singularity. That is, when we say the singularity is naked, we mean it is locally naked (i.e. after escaping through the apparent horizon, light rays may or may not be pass the event horizon).

It has been shown in Ref. [12] that the shell focusing singularity occurring at $r>0$, $R=0$ is totally space like, therefore we discuss about the central singularity only.

4 dimensional case of this type of models has been discussed in Ref.[13] and it has been shown that the gravitational collapse would ends in to a naked singularity

if

$$\lambda_0 \leq 0.1809, \tag{18}$$

while for $\lambda_0 > 0.1809$, the collapse leads to a black hole. Thus from Eqs. (17) and (18), one may argue that, there is a range of λ_0 :

$$0.1809 < \lambda_0 < 1, \tag{19}$$

in which the central singularity forms earlier than apparent horizon but it is not naked. We shall call this range as a 'trapped range' because it is a range of initial data in which the central singularity remains trapped though it forms earlier than apparent horizon. This is possible because even though there is no apparent horizon, an event horizon might well exists and an event horizon may clothe the singularities even if apparent horizon does not appears on the spatial slice considered [14]. In the case of 5 dimensional spacetime it has been shown in Ref. [11] that the gravitational collapse ends into a NS if $\lambda_0 \le 0.0901$, while it leads to a black hole if $\lambda_0 > 0.0901$. Thus in the *5D* case, 'trapped range' of initial data is given by

$$0.0901 < \lambda_0 < 1 . \tag{20}$$

Referring the critical values of λ_0 for the different higher dimensional space times from Ref. [11], we give the 'trapped ranges' of initial data for different higher dimensional spacetime as shown in the table I.

Table I

D	Critical values for *NS/BH*, λ_0	Trapped ranges of initial data	Naked singularity ranges of initial data
4	0.180916	0.180916 – 1	0 – 0.180916
5	0.901699	0.09016 – 1	0 – 0.09016
6	0.056372	0.056372 – 1	0 – 0.056372
7	0.039372	0.039372 – 1	0 – 0.039372
8	0.018285	0.01828 – 1	0 – 0.01828

It can be observed from table I that as the dimensions of the spacetime increases, trapped ranges of initial data also increase at the same time NS ranges of initial data (range of initial data in which the central singularity is naked) decrease.

We have calculated the time interval '$t_{ah}(r) - t_s(0)$' *for* $\lambda = 0.01$ *and* 0.001 in the different higher dimensional spacetimes as shown in the table II.

It can be observed from table II, that for the same initial data, the time interval, '$t_{ah}(r) - t_s(0)$' decreases with the increase in dimensions of the spacetimes.

Table II

N	D	$t_{ah}(r) - t_s(0)$ (for $\lambda_0 = 0.01$)	$t_{ah}(r) - t_s(0)$ (for $\lambda_0 = 0.001$)
2	4	6.66 r	21.018 r
3	5	4.95 r	15.755 r
4	6	3.9138 r	12.609 r
5	7	3.2279 r	10.4816 r
6	8	2.743 r	8.963 r

Further it has been argued in Ref. [15] that delays in the formation of apparent horizon would increase the possibility of the naked singularity in the collapse. Hence, decrease in the interval '$t_{ah}(r) - t_s(0)$' would imply decrease in the naked singularity spectrum.

III. GRAVITATIONAL COLLAPSE FROM FINITELY DIFFERENTIAL INITIAL DATA

We consider the initial density profile as [16]

$$\rho(r) = \rho_0 + \frac{\rho_1 r}{\rho_1 r} + \frac{\rho_2 r^2}{2!} + \frac{\rho_3 r^3}{3!} + \dots \quad . \tag{21}$$

It follows from Eq. (4) that the function $F(r)$ becomes fixed once the initial density distribution $\varepsilon(0,r) = \rho(r)$ is given.
i.e.

$$F(r) = \frac{2}{N} \int \rho(r) r^N dr \; . \tag{22}$$

From Eqs. (21) & (23) we obtain the expression for $F(r)$ as

$$F(r) = F_0 r^{N+1} + F_1 r^{N+2} + F_2 r^{N+3} + \dots \quad , \tag{23}$$

where

$$F_n = \frac{2}{N} \frac{\rho_n}{n!(N+1+n)} \quad . \tag{24}$$

Here we consider those density functions which decrease as one moves away from the center, hence first non-vanishing derivative of the density at the center is negative. Inserting the expression for $F(r)$ into Eq. (10) and keeping only the leading order terms we get

$$t_{ah}(r) = t_s(0) - \frac{1}{(N+1)}\frac{F_n}{F_0^{3/2}} r^n - \frac{2}{(N+1)} F_0^{1/(N-1)} r^{(N+1)/(N-1)} \ , \tag{25}$$

where $t_s(0)$ is the time at which central singularity forms and it is given by

$$t_s(0) = \frac{2}{(N+1)\sqrt{F_0}} \ . \tag{26}$$

Usual 4 dimensional dust collapse has been analyzed in Ref. [6] and it has been shown that if the first non vanishing derivatives of the density at the center is either ρ_1 or ρ_2 , then $t_s(0)<t_{ah}(r)$ and if the first non-vanishing derivatives is ρ_3 then $t_s(0)<t_{ah}(r)$ if

$$\frac{F_3}{F_0^{5/2}} < -2 \ . \tag{27}$$

It is also known that [16] the central singularity is naked if

$$\xi = \frac{F_3}{F_0^{5/2}} \le -25.9904 \ , \tag{28}$$

and the collapse ends into a black hole if

$$\xi = \frac{F_3}{F_0^{5/2}} > -25.9904 \, . \tag{29}$$

Thus as it is explained in [6], there is a range of ξ

$$-25.9904 < \xi < -2 \ , \tag{30}$$

in which even though the central singularity forms earlier than apparent horizon it is not naked. It can be seen through some manipulation that, the inequalities (19) and (30) are equivalent. We show this as follows :

Replacing λ_0 by F_0 in the inequality (19), We find that

$$0.076951545 < F_0^{3/2} < 1 \tag{31}$$

i.e.

$$12.99519063 > \frac{1}{F_0^{3/2}} > 1 \tag{32}$$

i.e.

$$-25.9904 < \frac{-2}{F_0^{3/2}} < -2 \tag{33}$$

i.e.

$$-25.9904 < \frac{F_3}{F_0^{5/2}} < -2 \ . \tag{34}$$

Thus with the substitution $F_3 = -2F_0$ in (30) we find that the inequalities (19) and (30) are identical.

Next we consider $N = 3$

It can be observed from Eq. (25) that in this case, $t_s(0) < t_{ah}(r)$ if $\rho_1 < 0$ and if the first non-vanishing derivative of the density at the center is ρ_2, then $t_s(0) < t_{ah}(r)$ if

$$\frac{F_2}{F_0^{2}} < -2 \ . \tag{35}$$

It is also known that [9], in this case, the central singularity is naked if

$$\frac{F_2}{F_0^{2}} < -22.18033 \ . \tag{36}$$

Hence combining the inequalities (35) and (36) together we can define the trapped range of initial data in the *5D* case as

$$-22.18033 < \frac{F_2}{F_0^{2}} < -2 \ . \tag{37}$$

Again with the substitution $F_2 = -2F_0$ in (37) it can be observed that the inequalities (20) and (37) are also equivalent.

Next we consider $N \geq 4$

From Eq. (25), it can be seen that in this case, $t_s(0) < t_{ah}(r)$, if $\rho_1 < 0$, which is always satisfied, because of our assumption about the density function. Also it has been shown in Ref.[9] that for $N \geq 4$, the singularity is naked if the first non-vanishing derivative of the density at the center is negative, irrespective of its magnitude. Hence for $N \geq 4$, no trapped ranges of initial data (or phase transition) exist.

IV. CONCLUDING REMARKS

In Ref. [6] it has been shown that formation of the central singularity earlier than apparent horizon is not the necessary and sufficient condition for nakedness. Considering non-self-similar TBL models (which can be reduced to self-similar under certain condition), we have generalized this result to any higher dimensional spacetimes.

It is found that if one applies the same type of initial data to all HD space times, then the time interval '$t_{ah}(r) - t_s(0)$' decreases with the increase in the dimensions of the space times, which can be associated with the decreased in the naked singularity spectrum in the collapse [15].

We have analyzed the formation of apparent horizons in two different classes of TBL models. The difference between theses classes of models has been discussed in Ref.[9]. It has been found that in the case of first types of models, phase transition parameter for NS/BH exists in all higher dimensional space times. It is also known that these singularities are strong in the Tiplers' sense [17]. In the case of second types of models (where the initial data is finitely differentiable) existence of the phase transition parameter for NS/BH is found up to only five dimensions, beyond that no phase transition is required for nakedness of the singularity[9].

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REFERENCE :

[1] R. Penrose, *Riv. Nuovo Cim* **1** 252 (1969).

[2] D.M. Eardley and L. Smarr, *Phys. Rev.* **D 19**, 2239 (1979).
D. Christodolou, *Commun. Phys (London)* **93**, 171 (1984).
R.P.A.C. Newman, *Class. Quantum Grav.* **3**, 527 (1986).
A. Papapetrou, in A random walk in relativity and cosmology edited by N. Dadhich, J.K. Rao, J.V. Narlikar and C.V. Vishveshwara (Wiley, New York) 184-191 (1985).
K. Lake and T. Zannias, *Phys. Rev*. **D43**, 1798 (1991).

[3] P.S.Joshi and I. H. Dwivedi, *Phys. Rev*. **D47**, 5357 (1993).
A. Ori ant T. Piran, *Phys. Rev.* **D42**, 1068 (1990).
T. Harada, *Phys. Rev.* **D58**, 104015 (1998).
K.D.Patil and U. S. Thool, *International Journal of Modern Physics* D Vol.14No.5, 873-882 (2005).
K.D.Patil, R.V. Saraykar and S.H.Ghate, *Pramana, J. Phys.* **52**, 553 (1999).
Umpei Miyamoto, Hideki Maeda and Tomohiro Harada, gr-qc/0411100 (2004).
Hideaki Kudoh, T. Harada, and H. Iguchi, *Phys. Rev.* **D 62** 104016 (2000).
Filipe C. Mena, Brien C. Nolan, and Reza Tavakol, *Phys. Rev.* **D70**, 084030(2004).
T.P. Singh and Cenalo Vaz, *Phys. Rev.* **D61**, 124005 (2000).
Sergio M.C.V. Goncalves, Sanjay Jhingan and Giulio Magli, *Phys. Rev.* **D 65**, 064011 (2002).

[4] S.W. Hawking and G.F.R. Ellis, *The large scale structure of space time* (Cambridge University Press) 1973.

[5] A. Banerjee, A. Sil, and S. Chatterjee, *The Astrophysical journal*, **422**, 681-687 (1994).
Ujjal Debnath, Subenoy Chakraborty and John D. Barrow, *Gen. Relativ. Gravit.* **36** No. 2231 (2004).

Asit Banerjee, Ujjal Debnath and Subenoy Chakraborty, *International Journal of Modern Physics* D, Vol. **12**, No.7, 1255-1263 (2003).
Ujjal Debnath and Subenoy Chakraborti, Gen. Relativ. Gravit. Vol 36, No 6, 1243 (2004).
Rituparno Goswami and Pankaj S. Joshi, *Phys. Rev*. **D 69**, 104002 (2004).

[6] Sanjay Jhingan, P.S.Joshi and T.P.Singh, *Class.Quantum Gravit*. **13**, 3057-3067(1996).

[7] James Geddes, *Phys. Rev.* **D 65**, 104015(2002).
N. Mohammedi, *Class.Quantum Gravit*. **21**, 3505-3514 (2004).
Tetsuya Shiromizu, Kazuya Koyama, Sumitada Onda, and Takashi Torii, *Phys. Rev.* **D68**, 063506 (2003).

[8] A.Sil and S. Chatterjee, *Gen. Relativ. Gravit*. Vol **26**, No 10, 999 (1994).
Kishor Patil, *Pramana, J. Physics*, Vol **60**, No. 03, 423-431 (2003).
S.G.Ghosh and N. Dadhich, *Phys. Rev.* **D 64**, 047501 (2001).
S.G.Ghosh and N. Dadhich, *Gen. Relativ.Gravit.* **35**, 359 (2003).
K.D.Patil, S.H.Ghate and R.V. Saraykar, *Pramana, J.Phys.* **56**, 503 (2001).
K.D.Patil, S.H.Ghate and R.V. Saraykar, *Indian J. Pure Appl.Math.* **33**, 379 (2002).

[9] K.D.Patil, *Phys. Rev.* **D 67**, 024017 (2003).
[10] I.H. Dwivedi and P.S.Joshi, *Class.Quantum Gravit.* **9**, L69 (1992).
P.S.Joshi and T.P.Singh, *Gen. Relativ. Gravit*. **27**, 921 (1995).
P.S.Joshi and T.P.Singh, *Phys. Rev.* **D 51**, 6778 (1995).
[11] S.G. Ghosh and A. Beesham, *Phys. Rev.* **D 64**, 124005 (2001).
[12] D. Christodoulou, in Ref. [2] above.
[13] P.S.Joshi and T.P.Singh, *Phys. Rev.* **D 51**, 6778 (1995).
[14] Takeshi Chiba, *Phys. Rev.* **D 60**, 044003 (1999).
[15] P.S.Joshi, N. Dadhich and R. Maartens, *Phys. Rev.* **D 65**, 101501 (2002).
[16] T.P.Singh and P.S.Joshi, *Class.Quantum Gravit.* **13**, 559 (199
[17] F.J. Tipler, *Phys. Lett.* **64A**, 8 (1977).